CALCULUS - 2

February 5, 2015

MEET 2

NAME _____

DIRECTIONS: Place your answer to each question below in the answer column.
POINT VALUE: Questions 1-6 are one point each and questions 7 & 8 are each two points.

Time: 40 minutes

ANSWER COLUMN

1) If $f(x) = |x^2 - 4|$, $f'(-3) =$ _____.
a) 2 b) -2 c) 6 d) -6 e) 0

1)_____

2) If $f'(x)$ exists for all x and $f(-2) = -10$ and $f(3) = 15$, then for at least one value of c where $-2 < c < 3$, what must be true?
a) $f'(c) = 25$ b) $f(c) = 25$ c) $f'(c) = 5$ d) $f(c) = -25$ e) $f'(c) = -5$

2)_____

3) A particle moves along the x-axis with its position , x , at time , t , given by $x = \frac{3t}{3+t}$. Its acceleration at $t = 0$ is _____.
a) -2 b) 3 c) -3 d) $\frac{2}{3}$ e) $\frac{-2}{3}$

3)_____

4) For a sphere, $V = \frac{4}{3}\pi r^3$ and (surface area) s.a. $= 4\pi r^2$. $\frac{dV}{dt} =$ _____.
a) $r\frac{d(s.a.)}{dt}$ b) $2r\frac{d(s.a.)}{dt}$ c) $12r\frac{d(s.a.)}{dt}$ d) $\frac{r}{2}\frac{d(s.a.)}{dt}$ e) $\frac{r}{4}\frac{d(s.a.)}{dt}$

4)_____

5) The function f(x) passes through the point (2,9) and at that instant has a slope of -1. If $g(x) = (f(x))^2$, then $g'(2) =$ _____.
a) 18 b) -18 c) 9 d) -9 e) 3

5)_____

6) The slope of the tangent to the curve $y = 4xy^2 - 5x + 3$ at the point where $y = 1$ is _____.
a) $\frac{3}{17}$ b) $\frac{1}{17}$ c) $-\frac{1}{17}$ d) $\frac{1}{15}$ e) $-\frac{1}{15}$

6)_____

7) The acceleration of a particle is given as a function of time by $a = 24t^2 - 8$. The position of the particle at any time , t , is s(t). If $s(1) = 5$ and $s(2) = 28$, find the velocity of the particle at $t = 2$.

7)_____

8) Chord $\overline{AB}$ of parabola $y = 4x^2$ moves up from the vertex at 2 units/sec., always staying parallel to the x-axis. Triangle ABC is formed by $\overline{AB}$ and the two tangents to the parabola, one at point A and one at point B. The triangle increases in area as $\overline{AB}$ moves up. The rate at which the area is increasing at the instant $\overline{AB}$ is 8 units above the vertex is _____ units^2/sec.

8)_____

SAMPLE

CALCULUS -2

MEET 2 **February 5, 2015** **CALCULUS ANSWER SHEET**

1) (d) $f(x) = \begin{cases} x^2 - 4 & \text{if } x \geq 2 \text{ or } x \leq -2 \\ 4 - x^2 & \text{if } -2 < x < 2 \end{cases}$. For $x = -3$, $f'(x) = 2x$. $f'(-3) = -6$.

2) (c) By The Mean Value Theorem, $f'(c) = \dfrac{f(b) - f(a)}{b - a} = \dfrac{15 + 10}{3 + 2} = 5$.

3) (e) $x = \dfrac{3t}{3 + t}$; $v = \dfrac{(3 + t)3 - 3t(1)}{(3 + t)^2} = \dfrac{9}{(3 + t)^2}$. $a = \dfrac{-18}{(3 + t)^3}$. At $t = 0$, $a = \dfrac{-2}{3}$.

4) (d) $V = \dfrac{4}{3}\pi r^3$; $\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$. $s.a. = 4\pi r^2$; $\dfrac{d(s.a.)}{dt} = 8\pi r \dfrac{dr}{dt}$. $\dfrac{dr}{dt} = \dfrac{1}{8\pi r} \dfrac{d(s.a.)}{dt}$.

$\dfrac{dV}{dt} = 4\pi r^2 \left(\dfrac{1}{8\pi r} \dfrac{d(s.a.)}{dt} \right) = \dfrac{r}{2} \dfrac{d(s.a.)}{dt}$.

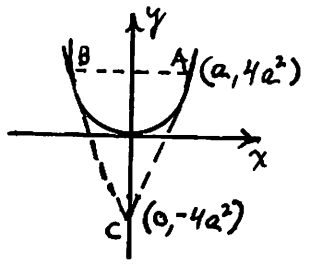
5) (b) $g(x) = (f(x))^2$. $g'(x) = 2f(x)f'(x)$. $g'(2) = 2f(2)f'(2) = 2(9)(-1) = -18$.

6) (d) $y - 4xy^2 + 5x = 3$. When $y = 1$, $x = 2$. $y' - 4y^2 - 8xyy' + 5 = 0$. $y' = \dfrac{4y^2 - 5}{1 - 8xy}$.

At $(2,1)$, $y' = \dfrac{1}{15}$.

7) (53) $a = 24t^2 - 8$; $v = 8t^3 - 8t + c_1$; $s = 2t^4 - 4t^2 + c_1 t + c_2$. $5 = 2 - 4 + c_1 + c_2$;
$c_1 + c_2 = 7$. $28 = 32 - 16 + 2c_1 + c_2$; $2c_1 + c_2 = 12$, $c_1 = 5$.
$v = 8(8) - 8(2) + 5 = 53$.

8) ($6\sqrt{2}$) $y = 4x^2$; $y' = 8x$. At any point $(a, 4a^2)$, the equation of the tangent line is $y = 8ax - 4a^2$. Point C has coordinates $(0, -4a^2)$. For any positive value of x, the length of the base is $2x$ and the altitude is $8x^2$.



$y = 4x^2$; $\dfrac{dy}{dt} = 8x \dfrac{dx}{dt}$, when $y = 8$, $x = \sqrt{2}$. $2 = 8\sqrt{2} \dfrac{dx}{dt}$;

$\dfrac{dx}{dt} = \dfrac{\sqrt{2}}{8}$. The area of $\triangle ABC = \dfrac{1}{2}(2x)(8x^2) = 8x^3$. $\dfrac{dA}{dt} = 24x^2 \dfrac{dx}{dt}$.

$\dfrac{dA}{dt} = 24(\sqrt{2})^2 \dfrac{\sqrt{2}}{8} = 6\sqrt{2}$ units^2/sec.